Functions & Proof Higher

What do I need to be able to do?

By the end of this unit you should be able to:

- Show that a solution to a complex equation lies between two given values
- Use an iterative formula to find approximate solutions to equations
- Understand the meaning of a function
- Know and use the notation for composite functions
- Find the inverse of a given function

Vocabulary

Iteration: repeating a process. Some questions can be answered by getting closer and closer using the same process each time.
Approximate: an estimation of a number
Solution: a value we can put in place of a variable that makes the equation true
Function: It is like a machine that has an input and an output
Composite function: a function made of other functions, where the output of one is the input to the other.
Inverse function: the reverse of another function.
Proof: logical mathematical arguments used to show the truth of a mathematical statement
Congruent: the same size and shape
Corresponding: items that appear in the same place in two similar shapes

Functions

A function links an input value to an output value. Functions are written in function notation with the name of the **function** (usually f or g), a **variable** written in brackets and an **expression**. When calculating the value of a function, the input value is substituted into the expression.

Example

$f(x) = 3x + 2$ and $g(x) = x^2 - 1$

Find $f(-2)$ and $g(3)$

$f(-2) = 3 \times -2 + 2 = -4$

$g(3) = 3 \times 3 - 1 = 8$

Inverse functions

A function links an input value to an output value. The inverse of a function is a function that links the output value back to the input value. The inverse function for $f(x)$ is written as $f^{-1}(x)$.

To find an inverse function, form an equation by giving the output value a name using a letter (such as y), then rearrange the equation to make x the subject.

Example

$f(x) = 5x - 4$

Find $f^{-1}(x)$

Form an equation by making $y = f(x)$ $y = 5x - 4$

Make x the subject. First, add 4 to both sides of the equation:

$y + 4 = 5x$

Then divide both sides by 5:

$\frac{y+4}{5} = x$

Finally, re-write the expression that is equal to x, replacing the y with an x:

The inverse function of $f(x) = 5x + 4$ is $f^{-1}(x) = \frac{x+4}{5}$

Composite functions

Composite functions are made when the output from one function is used as the input of another function. The names of the functions are written next to each other, with the function that is used first written next to the variable in brackets. The composite function $fg(x)$ means work out $g(x)$, then use this value in the function $f(x)$. So $fg(x) = f[g(x)]$

Example

$f(x) = 2x + 3$ and $g(x) = x^2$

Find $fg(4)$, $gf(4)$ and $ff(4)$

$fg(4)$ means work out $g(4)$, then work out $f(x)$ for this value.

$fg(4) = f[g(4)]$

$g(4) = 4^2 = 16$

so $f[g(4)] = f(16)$

$= 2 \times 16 + 3$

$= 35$

$gf(4)$ means work out $f(4)$, then work out $g(x)$ for this value

$f(4) = 2 \times 4 + 3 = 11$

So $gf(4) = g[f(4)] = g(11) = 11^2 = 121$

$ff(4)$ means work out $f(4)$, then work out $f(x)$ for this value

$f(4) = 2 \times 4 + 3 = 11$

So $ff(4) = f[f(4)] = f(11) = 2 \times 11 + 3 = 25$

Iteration

Approximate solutions to more complex equations can be found using a process called iteration. Iteration means repeatedly carrying out a process. To solve an equation using iteration, start with an initial value and substitute this into the iteration formula to obtain a new value, then use the new value for the next substitution, and so on.

Example:

When $x_0 = 0$ $\quad x_{n+1} = \frac{2}{x_n^2 + 3}$

Calculate the values of x_1, x_2, x_3 to find an estimate for the solution to $x^3 + 3x = 2$

$x_{0+1} = \frac{2}{0^2 + 3} = 0.\dot{6}$

We substitute this value into the next step.

$x_{1+1} = \frac{2}{0.\dot{6}^2 + 3} = 0.5806451613$

$x_{2+1} = \frac{2}{(0.58\ldots)^2 + 3} = 0.5993140006$

An estimate of the solution is 0.6 because all of the solutions round to 1d.p.