Formula – Y11 Higher – Unit 3

What do I need to be able to do?

By the end of this unit you should be able to:

- Add and subtract algebraic fractions
- Multiply and divide algebraic fractions
- Simplify an algebraic fraction
- Expand the product of three binomials
- Identify when factorisation is needed to simplify an algebraic fraction
- Simplify an algebraic fraction involving factorisation
- Change the subject of a formula when more than two steps are required
- Change the subject of a formula when the required subject appears twice

Vocabulary

Substitution: Replace the letter with a given value
Formula: expresses the relationship between two or more unknown values
Subject: The variable of a formula that is being calculated
Re-arrange: To change the position or order of something
Fractional: Expressed as or contains a fraction
Positive: A number greater than zero
Negative: A number less than zero
Factorisation: A way of writing an expression as a product of its factors, using brackets.

Writing a formula

A plumber has a call out fee of £40, plus an hourly rate of £18

With this information, we can write a formula to calculate the price of any job.

$T = 40 + 18h$

- T: Total price of the job
- 40: Represents the £40 which is a fixed price
- $18h$: The hourly rate of £18
- h: The number of hours worked. (We substitute here)

£18 is multiplied by the number of hours worked. We then add this to £40 to give the total cost

Substituting into a formula (positive numbers)

The total cost of a plumbing job is given by the formula below.

$T = 40 + 18h$

T = total cost of the job
h = hours worked

We can substitute into this formula to find the cost of any job

If a job took 2 hours.....

$T = 40 + 18h$
$T = 40 + (18 \times 2)$ — We substitute (swap) h for 2
$T = 40 + 36$
$T = 76$ — The total cost of the job is £76

Changing the subject of a formula (1-step)

We can re-arrange the formula to make 'a' the subject
(This means having 'a' on it's own on one side)

$a + 5 = c$

To get rid of the +5 we do the inverse operation which is -5. We must do this to both sides to balance the formula

$-5 \quad -5$

The +5 and -5 cancel to leave 'a' on its own

$a = c - 5$

We MUST do the same thing to both sides

Substituting into a formula (fractions)

The total cost of a plumbing job is given by the formula below.

$T = 40 + 18h$

T = total cost of the job
h = hours worked

Multiplying by ½ is the same as dividing by 2

If a job took ½ an hour.....

$T = 40 + 18h$
$T = 40 + (18 \times \frac{1}{2})$ — We substitute (swap) h for ½
$T = 40 + 9$
$T = 49$ — The total cost of the job is £49

Changing the subject of a formula (2-steps)

We can re-arrange the formula to make 'a' the subject

$2a + 5 = c$

STEP 1 Take 5 from both sides - *We do this first*

$-5 \quad -5$

$2a = c - 5$

STEP 2 Divide both sides by 2 – *This will mean 'a' is on its own as the subject of the formula*

$\div 2 \quad \div 2$

$a = \frac{c - 5}{2}$

We **MUST** do the **SAME** thing to both sides

Substituting into a formula (negatives)

The formula converts temperature from centigrade to Fahrenheit

$F = \frac{9}{5}C + 32$

F = temperature in Fahrenheit
C = temperature in centigrade

If the temperature is -10°C we substitute C for -10

$F = (\frac{9}{5} \times -10) + 32$ — We divide -10 by 5 and multiply by 9

$F = -18 + 32$
$F = 14°$

$-10 \div 5 = -2$
$-2 \times 9 = -18$

Negative ÷ positive OR Negative x positive = negative answer

Changing the subject of a formula (>2 steps)

$\frac{2a + 5}{7} = c$ — Multiply each side by 7 (Removes 7 from denominator)

$\times 7 \quad \times 7$

$2a + 5 = 7c$

STEP 2 Take 5 from both sides

$-5 \quad -5$

$2a = 7c - 5$

STEP 3 Divide both sides by 2

$\div 2 \quad \div 2$

$a = \frac{7c - 5}{2}$

A is now the subject of the formula

We **MUST** do the **SAME** thing to both sides

Changing the subject when it appears twice

$5a - 4b = 2a$

STEP 1 Take the smaller value of 'a' from both sides. There will now only be one value of 'a' on *the left*

$-2a \quad +4b \quad -2a \quad +4b$

STEP 2 Remove the other terms from the subject's side by doing the inverse operation

$3a = 4b$

$\div 3 \quad \div 3$

The 3's will cancel

$a = \frac{4b}{3}$

Divide both sides by 3 to make 'a' the subject

Adding Algebraic fractions

$\frac{2a}{9} + \frac{5a}{6}$

We multiply the one fraction by 2 and one by 3 to get a LCD of 18

$\frac{2 \times 2a}{2 \times 9} + \frac{3 \times 5a}{3 \times 6}$

To add any fractions, we must first have a Lowest Common Denominator (LCD). We must multiply the *WHOLE* fraction (top and bottom) to achieve this

$\frac{4a}{18} + \frac{15a}{18} = \frac{19a}{18}$

REMEMBER
We only add the numerators
The denominator stays the SAME

Subtract Algebraic fractions

$\frac{4a}{9} - \frac{a}{6}$

We multiply the one fraction by 2 and one by 3 to get a LCD of 18

$\frac{2 \times 4a}{2 \times 9} - \frac{3 \times a}{3 \times 6}$

To subtract any fractions, we must first have a Lowest Common Denominator (LCD). We must multiply the *WHOLE* fraction (top and bottom) to achieve this.

$\frac{8a}{18} - \frac{3a}{18} = \frac{5a}{18}$

REMEMBER
We only subtract numerators
The denominator stays the SAME

Multiply algebraic fractions

$\frac{3w}{5y} \times \frac{2y}{3w}$

METHOD 1 - Multiply the numerators and then the denominators (Like normal fractions)

$\frac{6wy}{15wy}$

METHOD 1 - Simplify the fractions and cancel

$\frac{6}{15} = \frac{2}{3}$

$\frac{6wy}{15wy}$

METHOD 2
Cancel numerators with any equivalent denominator in either fraction

$\frac{3w}{5x} \times \frac{2x}{3w}$

METHOD 2
You are then left with 2 as the numerator and 5 as the denominator

$\frac{2}{5}$

Dividing algebraic fractions

$\frac{a}{4} \div \frac{2a}{7}$

Just like a normal fraction we use KFC
Keep *(first fraction)*
FLIP *(second fraction)*
Change *(the ÷ to X)*

$\frac{a}{4} \times \frac{7}{2a}$

We then cancel and simplify! *(see multiplying fractions)*

$\frac{a}{4} \times \frac{7}{2a}$

$\frac{7}{4 \times 2}$

$\frac{7}{8}$

Simplify algebraic fractions (factorisation)

$\frac{4a^2 + 2ac}{6a + 3c}$

Factorise the numerator and denominator

$\frac{2a(a + c)}{3(a + c)}$

We can now cancel any terms common to the numerator and denominator. For this example it's $(a + c)$

$\frac{2a(a + c)}{3(a + c)}$

REMEMBER
We take out the Highest Common Factor (HCF) of each

$\frac{2a}{3}$

Expand the product of three binomials

$(a + 3)(a + 5)(a + 4)$

STAGE 1
1. Expand the first two brackets.
2. Collect like terms to simplify

$(a^2 + 5a + 3a + 15)$

$(a^2 + 8a + 15)(a + 4)$

STAGE 2
Multiply each term in bracket 1 by each term in bracket 2

$(a^3 + 4a^2 + 8a^2 + 32a + 15a + 60)$

$(a^3 + 12a^2 + 47a + 60)$

STAGE 3
Collect like terms

Notes