Proportional reasoning & Similar shapes H

What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise and interpret graphs that illustrate inverse & direct proportion
- Y
- Interpret equations that describe direct & inverse proportion
- Interpret equations that describe inverse proportion
- Solve inverse and direct proportion problems
- Identify congruence & similarity of shapes Finding missing lengths in similar shapes
- Know and use the criteria for triangles to be congruent (SSS, SAS, ASA, RHS)

Vocabulary

Direct Proportion: where the ratio between two quantities is a constant value. When the value of one variable increases, the other also increases

Inverse Proportion: the relationship between two variables. When the value of one variable increases, the other decreases, so their product is unchanged

Multiplier: the number you are multiplying by

Enlarge: to make a shape bigger by a given multiplier (scale factor)

Multiplier: the number you are multiplying by

Ratio: a statement of how two numbers compare

Similar: when one shape is an enlargement of another

Congruent: the same size and shape

Corresponding: items that appear in the same place in two similar shapes

Direct Proportion

As one variable changes the other changes at the same rate.

This is a multiplicative change

4 cans of pop = £2.40

4 cans of pop = £2.40 → ×3 → 12 cans of pop = £7.20

4 cans of pop = £2.40 → ×0.5 → 2 cans of pop = £1.20

This multiplier is the same In the same way that this would be for ratio

Sometimes this is easiest if you work out how much one unit is worth first e.g. 1 can of pop = £0.60

Inversely proportional is when one thing increases the other decrease

A teacher has 2 hours to mark some books.

Time spent	2	3	4	5	10	15	20
Number of books	60	40	30	24	12	8	6

×2 ×3 ÷2 ÷3

The number of books marked in 2 hours is inversely proportional to the time spent on each book.

Constant of proportionality

∝ is the symbol we use to show that one variable is in proportion to another.

Direct proportion $y \propto x$

g is directly proportional to the square root of h
When $g = 18$, $h = 16$
Find the possible values of h when $g = 2$

$g \propto \sqrt{h}$
$g = k\sqrt{h}$
$18 = k\sqrt{16}$
$18 = 4k$
$4.5 = k$
$g = 4.5\sqrt{h}$

$g = 4.5\sqrt{h}$
When $g = 2$
$2 = 4.5\sqrt{h}$
$\frac{2}{4.5} = \sqrt{h}$
$\left(\frac{4}{9}\right)^2 = h$
$\frac{16}{81} = h$

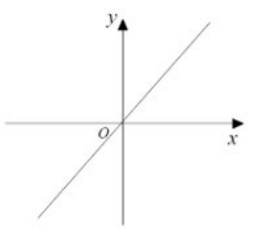
Inverse proportion $y \propto \frac{1}{x}$

The time taken, t, for passengers to be checked-in is inversely proportional to the square of the number of staff, s, working.
It takes 30 minutes passengers to be checked-in when 10 staff are working. How many staff are needed for 120 minutes?

$t \propto \frac{1}{s^2}$
$t = \frac{k}{s^2}$
$30 = \frac{k}{10^2}$
$3000 = k$
$t = \frac{3000}{s^2}$

$t = \frac{3000}{s^2}$
$120 = \frac{3000}{s^2}$
$s^2 = \frac{3000}{120}$
$s^2 = 25$
$s = \sqrt{25}$
$s = 5$

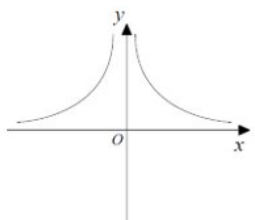
Direct and inverse proportion can also be represented on graphs:



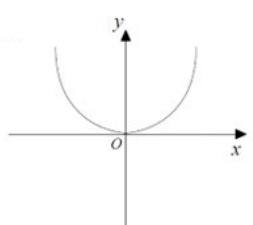
$y \propto x$

y is directly proportional to x

$y \propto \frac{1}{x}$



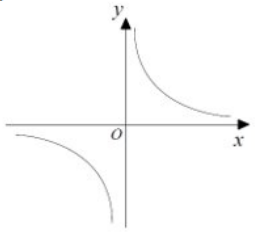
y is inversely proportional to x



$y \propto x^2$

y is directly proportional to x^2

$y \propto \frac{1}{x^2}$

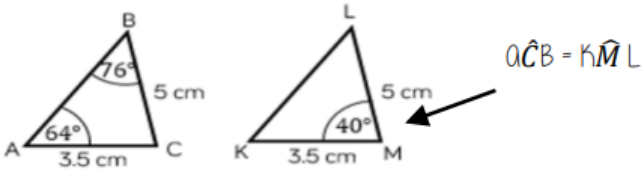


y is inversely proportional to x^2

Congruent figures

Congruent figures are identical in size and shape – they can be reflections or rotations of each other

Congruent shapes are identical – all corresponding sides and angles are the same size



$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**

Congruent triangles

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

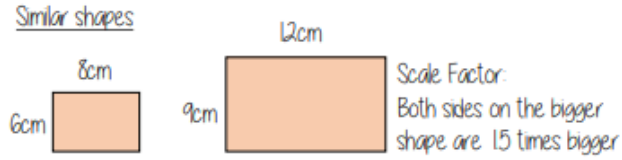
Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

Identify similar shapes

Angles in similar shapes do not change.
e.g if a triangle gets bigger the angles can not go above 180°

Similar shapes

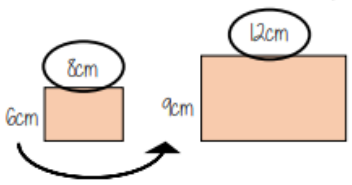


Scale Factor:
Both sides on the bigger shape are 1.5 times bigger

Compare sides: 6 : 9 = **2 : 3** 8 : 12 = **2 : 3**

Both sets of sides are in the same ratio

Information in similar shapes



x 1.5

Compare the equivalent side on both shapes

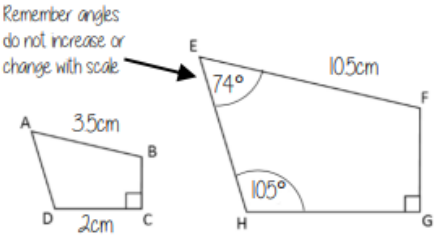
Scale Factor is the multiplicative relationship between the two lengths

Shape ABCD and EFGH are similar

Notation helps us find the corresponding sides

AB and EF are corresponding

Remember angles do not increase or change with scale



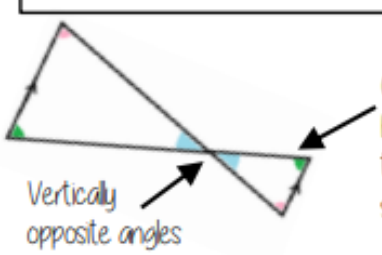
Similar triangles

Shares a vertex

Because corresponding angles are equal the highlighted angles are the same size

Parallel lines – all angles will be the same in both triangle

As all angles are the same this is similar – it only one pair of sides are needed to show equality

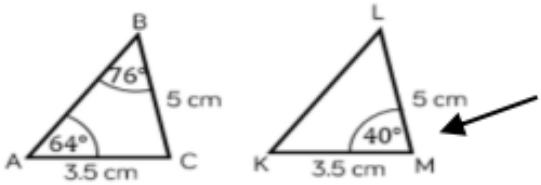


All the angles in both triangles are the same and so similar

Vertically opposite angles

Congruence and Similarity

Congruent shapes are identical – all corresponding sides and angles are the same size



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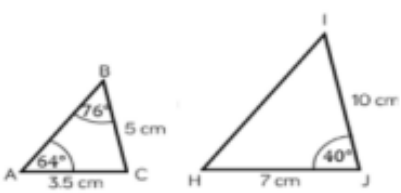

Because all angles are the same, but all sides are enlarged by 2 ABC and HIJ are **similar**