Proportional reasoning & Similar shapes F

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems involving ratio
- Solve problems with currency conversions
- Identify congruence of shapes in a range of situations
- Identify similarity of shapes in a range of situations
- Finding missing lengths in similar shapes
- Know and use the criteria for triangles to be congruent (SSS, SAS, ASA, RHS)
- Convert between units of length, area & volume

Vocabulary

Direct Proportion: where the ratio between two quantities is a constant value. When the value of one variable increases, the other also increases

Inverse Proportion: the relationship between two variables. When the value of one variable increases, the other decreases, so their product is unchanged

Multiplier: the number you are multiplying by

Enlarge: to make a shape bigger by a given multiplier (scale factor)

Multiplier: the number you are multiplying by

Ratio: a statement of how two numbers compare

Similar: when one shape is an enlargement of another

Compare with ratio (R)

"For every dog there are 2 cats"

Dogs: Cats

1:2

The ratio has to be written in the same order as the information is given.

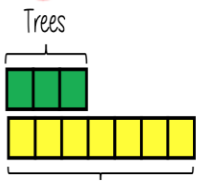
e.g. 2:1 would represent 2 dogs for every 1 cat

Units have the be of the same value to compare ratios

Ratios and fraction (R)

Trees: Flowers

3 : 7



Ratio

Fraction of trees

Number of parts of in group / Total number of parts = 3/10

Fraction

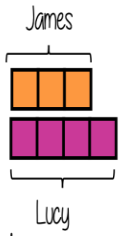
Sharing a whole into a given ratio (R)

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy

3 : 4



Find the value of one part

Whole: £350

7 parts to share between (3 James, 4 Lucy)

£350 ÷ 7 = £50

□ = one part = £50

Put back into the question

James: Lucy

x 50 3 : 4 x 50

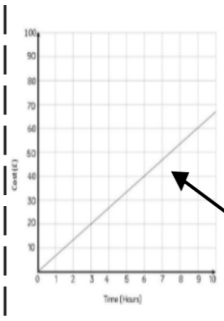
£150:£200

James = 3 x £50 = £150

£350

Lucy = 4 x £50 = £200

Ratio and graphs (R)



Graphs with a constant ratio are directly proportional

- Form a straight line
- Pass through (0,0)

The gradient is the constant ratio

Ratio and scale (R)

A picture of a car is drawn with a scale of 1:30

The car image is 10cm

Image : Real life

1cm : 30cm

10cm : 300cm

(x 10 / x 10)

Conversion between currencies (R)

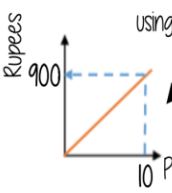
For every £1 I have 90 Rupees

£1 = 90 Rupees ← Currency is directly proportional

£1 = 90 Rupees (x 10 / x 10)

£10 = 900 Rupees

Currency can be converted using a conversion graph



Convert 630 Rupees into Pounds

£1 = 90 Rupees (x 7 / x 7)

630 ÷ 90 = 7

£7 = 630 Rupees

Ratios in 1:n and n:1

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

The question states that this part has to be 1 unit. Therefore Divide by 4

4 : 20

1 : 5

This side has to be divided by 4 too – to keep in proportion

the n part does not have to be an integer for this type of question

Length

mm (÷10 / x10) cm (÷100 / x100) m (÷1000 / x1000) km

10mm = 1cm
100cm = 1m
1000m = 1km
} REMEMBER!

Example 1

Convert 1.23m to mm

1.23m (x100) 123cm (x10) 1230mm

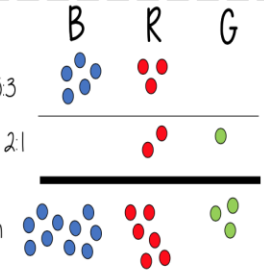
It is always helpful to break it up into stages. You could, of course, do this in one step!

Example 2

Convert 28400mm to km

28400mm (÷10) 2840cm (÷100) 28.4 (÷1000) 0.0284km

Combining ratios



The ratio of Blue counters to Red counters is 5:3

The ratio of Red counters to Green counters is 2:1

Ratio of Blue to Red to Green

10 : 6 : 3

Use equivalent ratios to allow comparison of the group that is common to both statements

Lowest common multiple of the ratio both statements share

Direct Proportion

As one variable changes the other changes at the same rate.

This is a multiplicative change

4 cans of pop = £2.40

4 cans of pop = £2.40 → 2 cans of pop = £1.20 (×0.5)

4 cans of pop = £2.40 → 12 cans of pop = £7.20 (×3)

This multiplier is the same in the same way that this would be for ratio

Sometimes this is easiest if you work out how much one unit is worth first e.g. 1 can of pop = £0.60

Inversely proportional is when one thing increases the other decrease

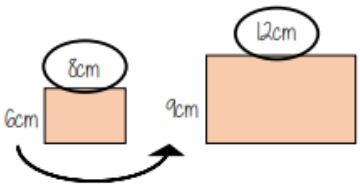
A teacher has 2 hours to mark some books.

Time spent	2	3	4	5	10	15	20
Number of books	60	40	30	24	12	8	6

×2 ×3 ÷2 ÷3

The number of books marked in 2 hours is inversely proportional to the time spent on each book.

Information in similar shapes



Compare the equivalent side on both shapes

Scale Factor is the multiplicative relationship between the two lengths

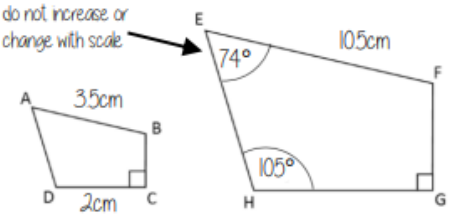
x 1.5

Shape ABCD and EFGH are similar

Notation helps us find the corresponding sides

AB and EF are corresponding

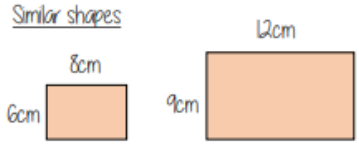
Remember angles do not increase or change with scale



Identify similar shapes

Angles in similar shapes do not change.
e.g. if a triangle gets bigger the angles can not go above 180°

Similar shapes



Scale Factor: Both sides on the bigger shape are 1.5 times bigger

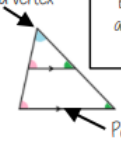
Compare sides: 6 : 9 = **2 : 3** 8 : 12 = **2 : 3**

Both sets of sides are in the same ratio

Similar triangles

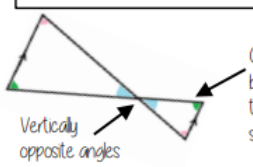
Shares a vertex

Because corresponding angles are equal the highlighted angles are the same size



Parallel lines – all angles will be the same in both triangle

As all angles are the same this is similar – it only one pair of sides are needed to show equality

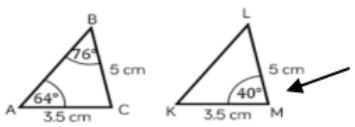


All the angles in both triangles are the same and so similar

Vertically opposite angles

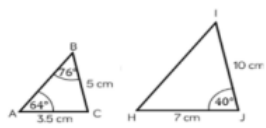
Congruence and Similarity

Congruent shapes are identical – all corresponding sides and angles are the same size



$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**



Because all angles are the same, but all sides are enlarged by 2 ABC and HIJ are **similar**

Conditions for congruent triangles

Triangles are congruent if they satisfy any of the following conditions

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

Notes